

Diese Verdamnte Quantenspringerei

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Abstract

It is argued that the conventional formulation of quantum mechanics is inadequate: the usual interpretation of the mathematical formalism in terms of the results of measurements cannot be applied to situations in which discontinuous transitions (“quantum jumps”) are observed as they happen, since nothing that can be called a measurement happens at the moment of observation. Attempts to force such observations into the standard mould lead to absurd results: “a watched pot never boils”. Experiments show both that this result is correct when the experiment does indeed consist of a series of measurements, and that it is not when the experiment consists of a period of observation: quantum jumps do happen. The possibilities for improving the formulation by incorporating transitions in the basic postulates are reviewed, and a satisfactory postulate is obtained by modifying a suggestion of Bell’s. This requires a distinction between the external description of the whole of a physical system and internal descriptions which are themselves physical events in the system. It is shown that this gives correct results for simple unstable systems and for the quantum-jump experiments.

1 Introduction

In 1929 Erwin Schrödinger complained “If I had known we were going to go on having all this damned quantum-jumping, I would never have got involved in the subject” [18]. Over twenty years later he was still not reconciled to the idea, posing the question “Are there quantum jumps?” as the title of a paper in the *British Journal for the Philosophy of Science*. After another 34 years the question was definitively answered in another title when Nagourney, Sandberg and Dehmelt published their paper “Shelved optical electron amplifier: observation of quantum jumps” in *Physical Review Letters*. But in spite of the direct and visible evidence for the reality of quantum jumps provided by this beautiful experiment, their place in theory is still not clear. The mathematical formulation of quantum mechanics is just as continuous as Schrödinger could have wished; the jumps, if they are present in the theory at all, are there only as part of the interpretive framework. In this paper I want to examine the necessity for and the possibility of an element of discontinuity in the theoretical framework of quantum mechanics.

To a particle physicist one of the most paradoxical aspects of quantum mechanics is that it is the only theoretical framework for one’s subject and yet it does not acknowledge the fundamental empirical elements in which one is interested. The evidence which must be explained by theories of elementary particle physics consists of *events* like the decay of the Ω^- particle captured in the bubble chamber photograph of Figure 1. But *there are no events in quantum theory*.¹ The nearest thing to an event described in the basic theory is the result of a measurement — something which has been deliberately provoked by an experimenter’s conscious action. There is no way that the theory can describe events, like the Ω^- decay in Figure 1, which happen spontaneously and are passively recorded by the waiting experimenter — which is how the photograph of Figure 1 was obtained.

Nevertheless, textbooks do purport to derive decay rates for such events from the general principles of quantum mechanics. The derivation goes like this. To describe a decay $A \rightarrow B + C$ we start at time $t = 0$ with a state $|A\rangle$ in which the unstable particle A is certain not to have decayed, and follow its time evolution, governed by a Hamiltonian H , to a superposition of the

¹The word “event” sometimes occurs in the sense of probability theory, with the meaning of “a statement being true” – for example, van Fraassen [37] claims to base his modal interpretation on events, but defines an event as the truth of a statement of the form “Observable B has value b ”. Throughout this paper I will use “event” with its primary English meaning of “happening”.

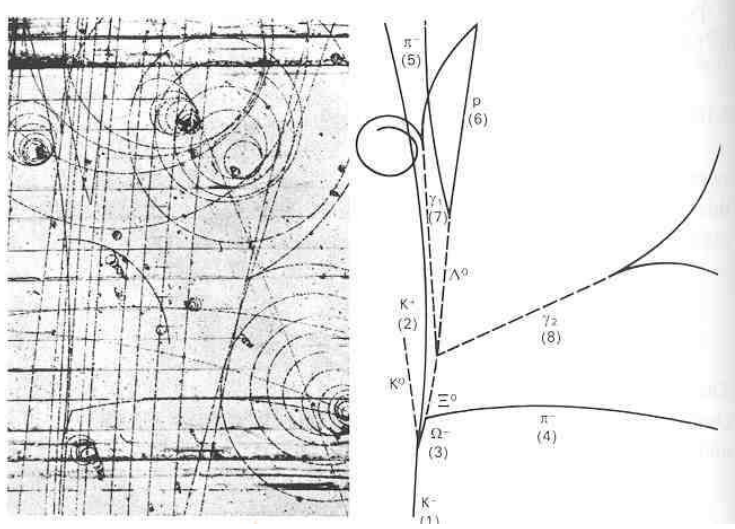


Figure 1: The world's first Ω^- event (Brookhaven National Laboratory, 1964).

initial state and a state of the decay products B and C :

$$e^{-iHt}|A\rangle = a(t)|A\rangle + b(t)|BC\rangle. \quad (1)$$

The general principles of quantum mechanics are then supposed to yield the interpretation that $|b(t)|^2$ is the probability that by time t there has been a transition from particle A to particles $B + C$.

But where does this notion of a “transition” come from? It appears nowhere in the general principles of the theory, as usually stated. The literal application of these principles to the state (1) gives only the statement that *if a measurement is made at time t* (to determine, say, whether a particle of type B is present), then $|b(t)|^2$ is the probability that the result of the measurement will be positive. The inference that something discontinuous (a transition) happened between time 0 and time t is completely unwarranted. According to the official principles, quantum systems evolve continuously (as the time dependence of (1) shows), and quantum jumps occur only when provoked by the intervention of an experimenter. Despite Schrödinger’s famous satire in his story of the cat, this remains the only description that the conventional form of the theory can tolerate; the idea of a transition must be a deplorable backsliding into classical habits of thought.

If this is what official quantum mechanics pronounces, then so much the worse for official quantum mechanics. In the actual physical situation that we are trying to describe, it is talk of “a measurement at time t ” that is

unwarranted. If a measurement can be said to take place at all when a decay is observed, it is an extended process occupying an interval of time rather than an instant; and the empirically meaningful time t is not the time at which the experimenter decides to make a measurement on the system, but the time at which the system does something for the experimenter to observe.

Of course, there is no questioning the success of the theory in predicting the right transition rates. The procedure that starts with the time-dependent state vector (1), produces from it a time-dependent probability $P(t)$, interprets this as a probability of something *having happened*, and on the strength of this interpretation derives from $P(t)$ a transition probability per unit time, certainly ends up with an empirically adequate result. The problem I want to discuss is that of formulating the theory so as to make this argument as sound as its conclusion. I will argue that the germ of a solution was provided by John Bell in 1984 ([6]; see also [34] and [9]), but that it needs to be transposed into a different interpretative framework in order to be fully satisfactory. In this paper I will do no more than suggest how this transposition might be achieved.

Before discussing Bell's formulation, however, I would like to sharpen the problem by reviewing (a) the result of taking the conventional interpretation literally and pushing it to what seems to be its logical conclusion, and (b) the experiments which demonstrate the reality of quantum jumps.

2 Quantum Jumps and Watched Pots: Theory and Experiment

A WATCHED POT NEVER BOILS

Let us imagine ourselves in the situation of steadily watching the unstable particle A in the process $A \rightarrow B + C$ — suppose it is a radioactive nucleus surrounded by a spherical detector wired to a loudspeaker, and we are listening for the click which will announce that the decay has occurred. Being attentive observers, we know at every instant of our watch whether we have heard a click or not. How can this situation be described in conventional quantum mechanics? It seems to require that a measurement is made at every instant, our knowledge at each instant being the result of the corresponding measurement. But this involves an uncountable number of measurements in a finite interval, which, as well as being embarrassing to analyse, probably flatters our vigilance. Let us compromise by considering a finite sequence of measurements and trying to capture the ideal of continuous

awareness by letting the measurements get closer and closer in time, taking the limit as the separation between them tends to zero. We will replace steady watching with repeated looking.

The result is disconcerting [42, 22, 33]. We suppose that in the time interval $[0, T]$ there are $N + 1$ measurements at times $0 = t_0, t_1, \dots, t_N = T$. Let p_n be the probability that the measurement at time t_n gives the result “A” (of the two possible results “A” or “B + C”), i.e. the probability that at this time we are aware that no decay has occurred. According to the projection postulate of conventional quantum mechanics, the measurement causes the system to jump into one of the states A or $B + C$, depending on the result of the measurement, after which it resumes (or rather — and this is crucial — *restarts*) its evolution following the Schrödinger equation. As a consequence of the unitary evolution in quantum mechanics, the resulting probabilities always satisfy:

Theorem [22, 33] In the limit as the dissection $\{t_0, \dots, t_N\}$ becomes infinitely fine, $p_N - p_0 \rightarrow 0$.

So if the unstable particle is observed assiduously enough, the probability that it has decayed at time T is the same as at time 0, namely zero if we start out knowing that it has not decayed. A watched pot never boils. Either that, or continual measurement is not a good model for *continuous* observation.

WATCHED POTS IN THE LABORATORY²

The conclusion of the above theorem has come to be known as the quantum Zeno paradox — an unfortunate name, since Zeno’s paradox of the arrow was resolved by Newton in classical mechanics and does not even arise in quantum mechanics. (Zeno’s argument was that if one only considers the configuration of a physical object in space, there is no difference between a stationary arrow and a moving one and therefore no reason why any arrow should behave differently from a stationary one. Newton’s answer is that position is not enough: at any instant, velocity is an independent property of an object which must also be specified before the equations of motion can be solved, these being second-order differential equations. In quantum mechanics, this does not apply. The equation of motion is first-order and a specification of configuration (in the form of a wave function $\psi(x)$) *is* enough. So Zeno’s paradox is purely classical, in physical terms as well as historical ones.)

The proverbial wisdom that a watched pot never boils, on the other hand, becomes an objective fact *only* in quantum mechanics. This is *not* because

²For a good recent review see [40].

of the role played by measurement — the significance of measurement in the proof of the watched-pot theorem is the change in the probabilities brought about by a change in knowledge, and this is as much a feature of classical probability as of quantum mechanics. The distinctive contribution of quantum mechanics to the calculation is the form of the transition probability $P(t)$ as a function of time. Because it is the square of the inner product between two vectors which are initially orthogonal, it behaves like t^2 for small t and is therefore slower to get started than a classical Poisson process, which behaves like t ; if it is continually forced to reset while in this vulnerable early stage of its life, it never gets off the ground at all.

Experiments can test this distinctive small-time behaviour and demonstrate the watched-pot phenomenon which can be uncontroversially deduced from it, without any appeal to dubious projection postulates, in a situation where there clearly are very short, rapidly repeated physical processes which can be treated as measurements. The experiments have been performed by Itano et al. [17] and confirm the theorem: the effect of very rapidly repeated, very short-duration interventions on an evolving quantum system is to freeze the evolution. This is the first arm of the disjunction at the end of the previous section. In the same and other laboratories the second arm of the disjunction has also been confirmed: if the experimental situation is one of passive, low-energy observation rather than active, high-energy probing, then the evolution is not affected, there is no watched-pot effect, and quantum jumps do occur.

In the quantum-jump experiment [25, 31, 8], originally proposed in 1975 by Dehmelt [14] as a practical method of detecting weak atomic transitions, a single trapped ion is observed as it makes transitions between three energy levels. The relevant levels, illustrated in Fig. 2, are a ground state and two excited states, one strongly coupled and one weakly coupled to the ground state. Let E_0 , E_1 and E_2 be the energies of the ground state, the strongly coupled excited state and the weakly coupled excited state, and let us suppose that the frequencies $(E_1 - E_0)/h$ and $(E_2 - E_0)/h$ are those of red and blue light respectively. The ion is illuminated by a laser beam tuned to the red frequency; absorption and stimulated emission of the laser mode cause the ion to oscillate between the ground state and the first excited state. Since these states are strongly coupled, the excited state also spontaneously emits photons at the red frequency but in other directions than that of the laser beam; these photons, which are emitted at a rate of thousands per second, enable the experimenter to see directly that the ion is in the subspace spanned by the ground state and the first excited state.

The ion is now illuminated by a second light source, low in intensity and tuned to the blue frequency, which can stimulate transitions from the

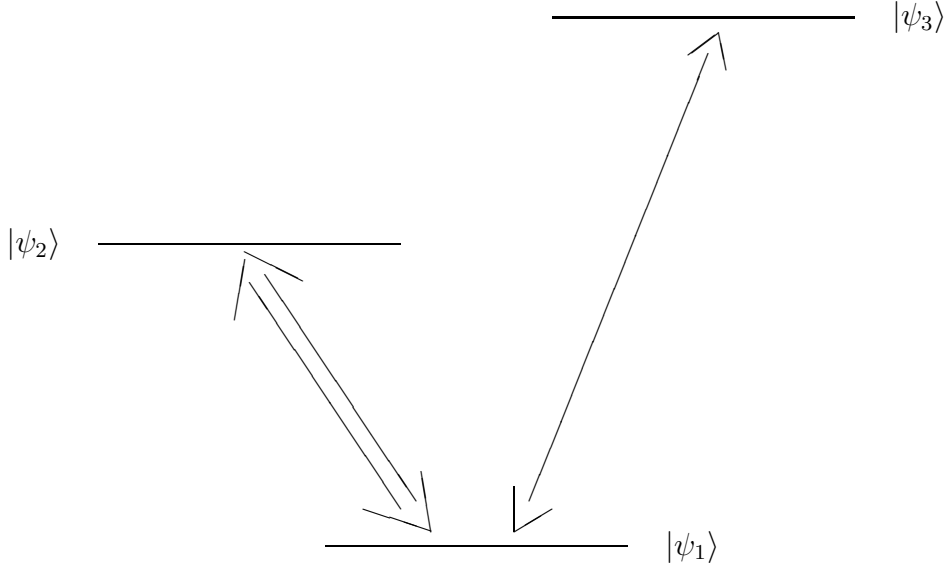


Figure 2: The atomic levels used in the quantum-jump experiment

ground state to the second excited state (the “shelf” state). Since this state is weakly coupled to the ground state, it has a long lifetime and the ion will stay on the shelf for some time. This is observed as an interruption in the red fluorescence from the first excited state. Seeing the sudden start of this dark period is an observation of a quantum jump from the ground state to the shelf; its equally sharp end marks a jump from the shelf down to the ground state.

The watched-pot experiment [17] uses a similar ion, but the roles of the two states are reversed. The continuous laser beam is now tuned to the blue frequency, so that the ion oscillates between the ground state and the weakly coupled second excited state; the weakness of the coupling makes the oscillation slow. This oscillation is regarded as the natural evolution of the two-state system formed by the ground state and the second excited state of the ion. The experiment consists of measurements to determine which of the two states the system is in, using intense laser pulses of light at the red frequency. If the ion is in the ground state, it will absorb a photon from the red pulse and be excited to the first excited state. Observation immediately afterwards of an emitted red photon, not in the direction of the laser beam, provides the result of the measurement (“ground state”) and, in accord with the projection postulate, leaves the system in the ground state. On the other hand, if the ion was in the second excited state then the red pulse will have no effect on it. Observation of no red photon provides the result “second

excited state” and leaves the system in that state.

In Itano and Wineland’s experiment a large number of such ions, initially in the ground state, were illuminated by the blue laser and allowed to oscillate without measurements for half a period. By switching off the blue laser beam and counting the photons emitted in the subsequent (slow) decay to the ground state, it was confirmed that the ions had all evolved as expected and reached the second excited state. Then the same evolution was subjected to repeated measurements by means of a large number of short red laser pulses. At the end of the same half-period, it was found that all of the ions were now in the ground state. The watched pot had not boiled.

WATCHED POTS IN THE STUDY

What is the theoretical difference between these two experiments, and why doesn’t the watched-pot theorem apply equally to both? In both cases we have an object system which is under investigation and whose state would, if left to itself, develop according to the Schrödinger equation:

$$|\psi_0\rangle \rightarrow |\psi(t)\rangle = a(t)|\psi_1\rangle + b(t)|\psi_2\rangle$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ are experimentally distinguishable states. We need to go deeper than the superficial interpretation which declares that a measurement at time t will have a result (1 or 2) and that after the measurement the system will be in the appropriate state $|\psi_1\rangle$ or $|\psi_2\rangle$. Following von Neumann, we must include the apparatus in the quantum description and consider a combined system of the object together with the apparatus, with product states $|\psi\rangle|\alpha\rangle$ where $|\alpha\rangle$ denotes a state of the apparatus. A measurement is brought about by an interaction between the object and the apparatus which changes the combined state as follows:

$$\begin{aligned} |\psi_1\rangle|\alpha_0\rangle &\rightarrow |\psi_1\rangle|\alpha_1\rangle \\ |\psi_2\rangle|\alpha_0\rangle &\rightarrow |\psi_2\rangle|\alpha_2\rangle \end{aligned}$$

where $|\alpha_0\rangle$ is the initial (“ready”) state of the apparatus and $|\alpha_1\rangle$ and $|\alpha_2\rangle$ are the states in which it registers the results 1 and 2. Thus the whole process of making a measurement on a changing object system can be represented as

$$|\psi_0\rangle|\alpha_0\rangle \rightarrow |\psi(t)\rangle|\alpha_0\rangle \rightarrow a(t)|\psi_1\rangle|\alpha_1\rangle + b(t)|\psi_2\rangle|\alpha_2\rangle.$$

It is clear from this that the Hamiltonian for the interaction between the apparatus and the object must be time-dependent; the apparatus has to be switched on at time t , when the measurement starts. Here is the difference

between the watched-pot experiment and the quantum-jump experiment. In the first the interaction between the object and the apparatus is indeed time-dependent; the apparatus consists of the pulsed laser beam, and there is time dependence in the pulsing. In the quantum-jump experiment there is no time dependence: there is a steady laser beam, and a photon detector which is switched on before the experiment begins and is in a constant state of readiness to respond. Thus the theoretical measurement events at times t_0, \dots, t_N in the watched-pot theorem fairly describe the physical reality in the watched-pot experiment, but they have no place in the quantum-jump experiment and it should be no surprise that the conclusion of the theorem is not verified.

A theoretical analysis [33] of continuous observation of an unstable system, with a constant interaction between the observing apparatus and the decaying system, gives some reason to believe in the watched-pot effect even in this situation. The crucial parameter is the response time of the observing apparatus in comparison to the time constants of the decaying system. It is not the lifetime T of the decay that is important, but the time τ during which the decay probability behaves quadratically, before the onset of the exponential regime which is all that is usually observed. If the apparatus responds much faster than τ , then its mere presence is enough to inhibit the decay. However, since the non-exponential decay time τ is several orders of magnitude smaller than the lifetime and has never been observed in real unstable systems, it is difficult to test this version of the watched-pot effect.

3 Transition Postulates

NECESSARY TRANSITION PROBABILITIES

We are left with a theoretical problem. If we cannot use the notion of “measurement at time t ” to explain the meaning of a time-dependent state vector, what can we say that will justify or codify what physicists do with such vectors, and that will predict our experience of quantum jumps? A simple answer is that transitions must be given a fundamental role in the theory; one of its basic postulates should be of the form “If the system [however broadly conceived] is in state ψ at time t , there is a probability $T_{\phi\psi}(t)dt$ that it will make a transition to state ϕ between t and $t + dt$ ”. Such a postulate, if it is to be fundamental, would need to be accompanied by a clear statement of exactly what the eligible states ϕ, ψ are. We certainly need something less *ad hoc* than the prescription students are left to deduce from current textbook paradigms, in which it is tacitly assumed that systems

make transitions between eigenstates of H_0 , a reference Hamiltonian which is chosen for no more principled reason than its convenience or familiarity.

A suggestion of this kind was made by John Bell in 1984 [6]. I will present it in a generalised form [34, 35, 9]. The basic idea is that there is a set of special physical quantities, which have a fundamental status; Bell liked to call them *beables* (as opposed to “observables”) to emphasise their objective nature (though the proposal is not just that they can be but that they do be). These quantities always have definite values, which change stochastically according to transition probabilities which are determined by the solution to the Schrödinger equation. Equivalently, one can replace the special quantities by a special set of subspaces of state space, namely their eigenspaces, and the actual values of the special quantities by a vector (the projection of the full state vector) in the corresponding subspace. I will call these subspaces the *viable* subspaces. They may vary with time.

Then the complete description of a physical system at any instant of time has two parts: a state vector in one of the viable subspaces, which is what we are aware of and which I will call the *visible* state; and the solution of the Schrödinger equation, in general a superposition of possible visible states, which guides the transition of the visible state between the different subspaces. I will call the latter the *pilot* state (following Bell [5], who used this term to describe the role of the wave function in the de Broglie/Bohm theory, a special case of this theory in which the visible states are eigenstates of position [35]).

POSSIBLE TRANSITION PROBABILITIES

We know how the pilot state changes in time: it satisfies the Schrödinger equation. What are the rules governing the change of the visible state? We will now, following Bacciagaluppi and Dickson [2], discuss the possible transition probabilities which are consistent with the usual quantum-mechanical rules for the results of measurements.

Let $|\Psi(t)\rangle$ be the pilot state, let $\mathcal{S}_1(t), \mathcal{S}_2(t), \dots$ be the viable subspaces at time t , and let $\Pi_m(t)$ be the projection onto \mathcal{S}_m . Then the visible state at time t is one of the states $|\psi_m(t)\rangle = \Pi_m|\Psi(t)\rangle$, and there is a set of positive real numbers T_{nm} such that for $m \neq n$, the probability of transition from $|\psi_m(t)\rangle$ at time t to $|\psi_n(t + \delta t)\rangle$ at time $t + \delta t$ is $T_{nm}\delta t$ (i.e. $T_{nm}\delta t$ is the probability that the visible state is $|\psi_n(t + \delta t)\rangle$ at time $t + \delta t$ if it is $|\psi_m(t)\rangle$ at time t). Let $P_m(t)$ be the probability that the visible state is $|\psi(t)\rangle$ at time t ; this then satisfies the master equation

$$\frac{dP_m}{dt} = \sum_n (w_{mn} - w_{nm}) \quad (2)$$

where $w_{mn} = T_{mn}P_n$.

We would like to identify $P_m(t)$ with the quantum-mechanical probability

$$P_m(t) = |\langle \Psi(t) | \psi_m(t) \rangle|^2 = |\langle \Psi(t) | \Pi_m(t) | \Psi(t) \rangle|^2 \quad (3)$$

which gives

$$\frac{dP_m}{dt} = 2 \operatorname{Re} \left[\langle \Psi(t) | \Pi_m(t) | \frac{d\Psi}{dt} \rangle + \langle \Psi(t) | \frac{d\Pi_m}{dt} | \Psi(t) \rangle \right]$$

Since the projectors Π_m are complete, this can be written as

$$\frac{dP_m}{dt} = \sum_n J_{mn}$$

where

$$J_{mn} = 2 \operatorname{Im} \langle \Psi | \left(\hbar^{-1} \Pi_m H \Pi_n + \frac{d\Pi_m}{dt} \Pi_n \right) | \Psi \rangle, \quad (4)$$

H being the Hamiltonian. Then the orthogonality $\Pi_m \Pi_n = \delta_{mn}$ gives

$$J_{mn} = -J_{nm}$$

so it is possible to find w_{mn} such that

$$w_{mn} - w_{nm} = J_{mn} \quad \text{and} \quad w_{mn} \geq 0.$$

One possible solution is

$$w_{mn} = \max(J_{mn}, 0) \quad (5)$$

but any positive symmetric x_{mn} could be added to this. More generally, x_{mn} need not be symmetric but must satisfy $\sum_n (x_{mn} - x_{nm}) = 0$.

The transition probabilities (5) were originally suggested by Bell [6] and extended to the time-dependent case by Bacciagaluppi and Dickson [2]. Vink [39] has investigated more general possibilities and shown that they include Nelson's stochastic dynamics [26]. It has been shown [35] that the Bell transition probabilities (5) yield the deterministic de Broglie-Bohm theory in the continuum limit of a lattice model of a particle moving in a potential. Bell's solution has the feature that between any given pair of states, the transitions are all one-way; if there can be a transition from $|\psi_m\rangle$ to $|\psi_n\rangle$, there cannot be a transition from $|\psi_n\rangle$ to $|\psi_m\rangle$. As we will now see, this is a natural feature of decay processes.

APPLICATION TO DECAY PROCESSES

Let us return to the problem of the unstable particle and model the decay $A \rightarrow B + C$ a little more realistically. If the decay products had just the one state $|BC\rangle$, the hermiticity of the hamiltonian would guarantee that the initial state $|A\rangle$ would be regenerated. For an irreversible decay of the kind we observe, the decay products must have an infinite-dimensional state space \mathcal{S}' in which they can disperse [36]. This makes it possible to suppose that the time of decay can be measured by examining the decay products. We can continue to suppose that the unstable particle A has just one state, coupled by the decay Hamiltonian to a decay state $|BC(0)\rangle \in \mathcal{S}'$ which then evolves by the internal dynamics of the BC system to $|BC(t)\rangle \in \mathcal{S}'$ after time t (as well as being coupled back to $|A\rangle$ by the interaction). Thus the Hamiltonian is $H = H_0 + \varepsilon H'$, where:

- (i) $|\psi_0\rangle$ is an eigenstate of H_0 ;
 - (ii) H_0 acts inside \mathcal{S}' to govern the dispersal of the decay products, so that $e^{-iH_0 t}|\psi_1\rangle$ rapidly becomes orthogonal to $|\psi_1\rangle$, and remains so:
- $$\langle \psi_1 | e^{-iH_0 t/\hbar} | \psi_1 \rangle = 0 \text{ unless } t < \tau; \quad (6)$$
- (iii) $H' = |BC(0)\rangle\langle A| + |A\rangle\langle BC(0)|$.

With this Hamiltonian, the solution of the Schrödinger equation with initial condition $|\psi(t)\rangle = |\psi_0\rangle$ at $t = 0$ is [33]

$$|\psi(t)\rangle = f(t)|\psi_0\rangle + \frac{\varepsilon}{i\hbar} \int_0^t f(t') e^{-iH_0(t-t')/\hbar} |\psi_1\rangle dt' \quad (7)$$

where the survival amplitude $f(t)$ is the solution of the integro-differential equation

$$\frac{d}{dt} [e^{-iE_0 t/\hbar} f(t)] = -\frac{\varepsilon^2}{\hbar^2} \int_0^t \chi(t-t') f(t') e^{iE_0 t'/\hbar} dt' \quad (8)$$

with

$$\chi(t) = \langle \psi_1 | e^{-iH_0 t/\hbar} | \psi_1 \rangle.$$

According to (6) the function $\chi(t)$ vanishes for $|t| < \tau$, so an approximate solution to (8), which neglects the variation of f on time scales of the order of τ , is

$$f(t) \approx e^{-\Gamma\tau} e^{-iE_0 t/\hbar}$$

with

$$\Gamma = \frac{\varepsilon^2}{\hbar^2} \int_0^\tau \chi(t) dt.$$

Now let us adopt Bell's interpretation with just two viable subspaces \mathcal{S}_0 and \mathcal{S}_1 , where \mathcal{S}_0 is the one-dimensional subspace spanned by $|\psi_0\rangle$ and \mathcal{S}' as above is the subspace containing the states of the decay products. This is to assume that at any time the system consists either of the unstable particle A or of a decayed state $B + C$, and it makes transitions between the two with probabilities given by (4) and (5). We would expect the transitions to go only in the direction $A \rightarrow B + C$.

Using the suffix d (= decayed) to refer to \mathcal{S}' , we have transition probabilities T_{0d} and T_{d0} determined by the real part of the matrix element

$$\frac{1}{i\hbar} \langle \psi(t) | \Pi_0 H \Pi_d | \psi(t) \rangle = -\frac{\varepsilon^2}{\hbar^2} \overline{f(t)} \int_0^t q(t-t') f(t') dt'.$$

At times $t \gg \tau$, this is approximately $-\Gamma |f(t)|^2$. Hence the transition probabilities have exactly their expected values: $T_{0d} = 0$ and $T_{d0} = \Gamma$.

There is still a possibly counter-intuitive element in this picture. If a transition occurs from the unstable state $|\psi_0\rangle$ at time t , it goes to the state

$$\Pi_d |\psi(t)\rangle = N^{-1} \int_0^t f(t') e^{-iH_0(t-t')/\hbar} |\psi_1\rangle dt'$$

(where N is a normalisation factor) which is a superposition of states which have decayed at earlier times $t' < t$. If this superposition is examined to determine the time t' of decay, there appears to be a possibility that the result of the measurement will not be the time t at which the transition occurred according to the stochastic theory, in which case the theory would not be giving an empirically verifiable account.

To investigate this, let us now consider a model with a more refined description of the visible states. If the decayed states in \mathcal{S}' can be examined to determine the time of decay to $|\psi_1\rangle$, then $e^{-iH_0 t/\hbar} |\psi_1\rangle$ evolves through a sequence of eigenstates $|\psi_n\rangle$ of an observable T = "time since decay". Suppose that the eigenvalues of T are spaced at equal intervals τ ; then (6) is generalised to

$$\langle \psi_n | e^{-iH_0 t/\hbar} | \psi_m \rangle = 0 \text{ unless } |t - (n - m)\tau| < \tau. \quad (9)$$

Now the transition probability from $|\psi_0\rangle$ to $|\psi_m\rangle$ is the imaginary part of

$$\langle \psi(t) | \Pi_m H \Pi_0 | \psi(t) \rangle = \varepsilon f(t) \langle \psi(t) | \Pi_m | \psi_1 \rangle$$

which vanishes unless $m = 1$. Thus the decay can only be to the correct state $|\psi_1\rangle$. This confirms that, if the viable subspaces are chosen as above, Bell's transition probabilities give a satisfactory description of the decay process.

CHOICES, CHOICES ...

That last sentence would be a triumphant conclusion if it were not for the fatal weakness of its qualifying clause. Bell's formulation of quantum mechanics does not exist until one has specified the viable subspaces. In this respect it is no improvement on the conventional formulation, which does not exist until one has specified precisely what physical arrangements constitute "measurements". The simplest approach to the problem in the case of Bell's formulation is to specify a preferred set of observables – sorry, beables – whose eigenspaces will be the viable subspaces. However, this is bound to seem arbitrary; if one is aspiring to give an absolute description of the physical world, there seems to be no good empirical or theoretical reason why any particular set of physical quantities should have fundamental status. Bell's own proposal in [6] was that these quantities should be the bilinear invariants of the fundamental fermion fields, but this only emphasises the arbitrariness: to pick out a particular set of fermion fields as "fundamental" abolishes the freedom to make field redefinitions, which one would like to regard as yielding equivalent theories.

A related problem is that of relativistic invariance. A theory based on transitions between states of the world requires these states to be defined universally at each instant of time, and therefore has a definition of simultaneity built in; in other words, it has a preferred frame of reference. Since the theory makes the same predictions as conventional relativistic quantum field theory, this frame of reference would not be experimentally distinguishable from any other frame related to it by a Lorentz transformation, but as Bell conceded in the final sentence of [6], "It seems an eccentric way to make a world."

There is a way of avoiding the arbitrariness of a choice of preferred subspaces if the world can be divided into two subsystems, so that the pilot state space has the structure of a tensor product $\mathcal{S}_1 \otimes \mathcal{S}_2$. In that case any vector in the tensor product canonically defines a decomposition of the space into orthogonal subspaces by means of the Schmidt decomposition: given $|\Psi(t)\rangle \in \mathcal{S}_1 \otimes \mathcal{S}_2$, we have

$$\mathcal{S}_1 \otimes \mathcal{S}_2 = \sum_{\lambda} \mathcal{S}_{1\lambda}(t) \otimes \mathcal{S}_{2\lambda}(t) \oplus \mathcal{S}'$$

where $\mathcal{S}_{1\lambda}(t) \subset \mathcal{S}_1$ is the eigenspace of the marginal density matrix $\rho_1(t) = \text{tr}_2 |\Psi(t)\rangle\langle\Psi(t)|$ with eigenvalue λ , $\mathcal{S}_{2\lambda}(t) \subset \mathcal{S}_2$ is the eigenspace of $\rho_2(t)$ with the same eigenvalue, and \mathcal{S}' is the residual subspace of $\mathcal{S}_1 \otimes \mathcal{S}_2$, orthogonal to all the $\mathcal{S}_{1\lambda} \otimes \mathcal{S}_{2\lambda}$. Taking the viable subspaces at time t to be the

$\mathcal{S}_{1\lambda}(t) \otimes \mathcal{S}_{2\lambda}(t)$ and \mathcal{S}' gives a formulation in which the subsystem S_1 is always in an eigenspace of its density matrix, and the rest of the world is in a corresponding state.

Such formulations of quantum mechanics, based on the Schmidt decomposition in a given tensor product structure, are a subclass of “modal interpretations” [37, 15, 2]. They offer the prospect of a theory which describes real events, reflecting our actual experience of the world, without compromising the unitary symmetry of quantum mechanics. Unfortunately, the Schmidt decomposition has not proved true to its promise that it would deliver a canonically defined set of visible states. It may be granted that the world consists of subsystems and therefore its state space has a tensor product structure (though it might also be queried whether this decomposition is uniquely given as part of nature), but it is quite clear that there are more than two subsystems. It is only in a tensor product of two factors, however, that vectors are guaranteed to have a Schmidt decomposition of the kind used in modal interpretations; in an n -fold tensor product $\mathcal{S}_1 \otimes \cdots \otimes \mathcal{S}_n$ it is rather unusual [29] for vectors to be of the form

$$\sum_i^N c_i |\psi_i^{(1)}\rangle \cdots |\psi_i^{(n)}\rangle$$

where $|\psi_i^{(r)}\rangle (i = 1, \dots, N)$ are orthonormal vectors in \mathcal{S}_r . (There is a generalisation of the Schmidt decomposition which is valid for any vector in an n -fold tensor product [11], but it does not relate to the marginal states of the subsystems as the bipartite Schmidt decomposition does.) In order to take advantage of the properties of the Schmidt decomposition, therefore, the theory must specify the way that it divides the world into one particular subsystem and the rest of the world. We find ourselves once again having to make arbitrary choices. Desperate attempts to have our cake and eat it are doomed to failure [38].

... AND HOW TO LIVE WITH THEM

If arbitrary choices cannot be avoided, we must consider how to live with them. But we are used to living with arbitrary choices in physics; they are characteristic of relativistic theories. There is nothing objectionable about being forced to make an arbitrary choice as long as we recognise that our descriptions are made relative to that choice, that any other choice would be equally valid, and as long as we know how to change from one such choice to another.

We have just been considering arbitrary choices of preferred subspaces, preferred physical quantities or preferred decompositions of the world into subsystems. They seem objectionable if they are seen to form part of an objective or absolute description of the world, seen from outside. But there can be no objection to preferred subspaces, observables or subsystem occurring in a description of the world from inside the world. Then, indeed, one is obliged to say from where one is observing, what subsystem of the world is doing the observing, and what observables are being observed.

This brings us to Everitt's relative-state formulation of quantum mechanics, which is closely related to Bell's beable formulation and to modal interpretations, though in some respects it is directly opposed to them. I would now like to present the relative-state formulation in a version which emphasises these similarities and differences, while also relating it to the conventional interpretation. I hope this will support a claim that it is the natural context in which to introduce the quantum jumps whose necessity is the subject of this paper.

INTERNAL AND EXTERNAL

In Bell's beable interpretation the identities of both the pilot state and the visible state are, at any given time, facts about the world. They have the same propositional status. In the modal interpretation an identification of the pilot state is a modal statement: it tells us what can, or must, or might be the case. What *is* the case is the identity of the visible state. In Everett's relative-state interpretation the identities of both the pilot state and the visible state are facts, but they are facts of different kinds. The difference can be described by the pairs of words "absolute" and "relative", or by "objective" and "subjective". I prefer to use "external" and "internal". Most vividly, the distinction has been described by Nagel [24] as that between "the view from nowhere" and "the view from now here".

As always, the best system to illustrate the principle is Schrödinger's cat, with an unstable atom, a diabolical device and a cat Jacko in a locked box made of unbreakable glass so that the human observer is helplessly aware of the moment of the cat's death. If the atom is in its excited state at time $t = 0$, the state of our little universe at time t is given by a vector in the tensor product $\mathcal{S}_{\text{atom}} \otimes \mathcal{S}_{\text{cat}} \otimes \mathcal{S}_{\text{human}}$ of the form

$$|\Psi(t)\rangle = e^{-\gamma t} |\text{excited}\rangle |\text{alive}\rangle |\text{"Jacko is alive"}\rangle \\ + \int_0^t e^{-\gamma t'} |\text{ground}\rangle |\text{dead for time } t - t'\rangle |\text{"Jacko died at } t'\rangle dt'. \quad (10)$$

The statements inside the ket signs for the vectors in $\mathcal{S}_{\text{human}}$ indicate brain states in which those statements occur as beliefs. They are physical configurations; but they are also statements. Are they true or false? Each is believed by a brain which has observed the fact it describes, and that fact belongs to reality. As a human belief, each statement could not be more true. Yet they cannot both be true, for they contradict each other. I take this to be characteristic of *internal* statements in a physical system; the belief of such a statement is a physical occurrence, and its truth can only be assessed in the physical context in which it occurs. In the present situation, such a context consists of a particular component of the universal state (10).

Internal statements are necessarily part of the experience of a conscious physical being (I use the word “conscious” reluctantly, because I do not want to be understood as restricting the discussion to human beings, but it seems to be what I mean). But they are not (or not solely) *about* experiences – on the contrary, they are about the physical world, most interestingly the part of the world which is separate from the experiencing being. What makes them internal statements is that they are themselves events in the physical world.

The distinction between internal and external statements offers a way to resolve the tension between the two kinds of time development in the conventional textbook formulation of quantum mechanics: the continuous, deterministic evolution of the state vector given by the Schrödinger equation, and the discontinuous, probabilistic change following a measurement, given by the projection postulate. The idea, roughly speaking, would be that the Schrödinger equation is an external statement, while the projection postulate is an internal one. More precisely, let $|\Psi(t)\rangle$ be a time-labelled sequence of states satisfying the Schrödinger equation. Then it is an external (and tenseless) statement that the world passes through the sequence of states $|\Psi(t)\rangle$. Suppose these states are products $|\phi(t)\rangle|\psi(t)\rangle$ where $|\phi(t)\rangle$ is a state of a measuring apparatus and a conscious observer, while $|\psi(t)\rangle$ is the state of the rest of the world (or simply the system being measured by the apparatus), and suppose the Hamiltonian includes a measurement (an instantaneous one, since I am — for the moment — in that convention) made at time t_0 . Then it is an internal statement (and a tensed one, with a “now” of $t_0 + \epsilon$) that after the measurement the state $\phi(t)$ has jumped to an eigenstate of the measured quantity. A report of a measurement result is an internal statement, made by a conscious system; a general statement about measurement results must be made relative to a particular conscious system (just as statements about energy in special relativity are necessarily relative to a particular frame of reference).

If the argument of this paper is accepted, statements about results of

measurements should be replaced by statements about spontaneous transitions, or quantum jumps. But these should still be statements of the same type: a report of a quantum jump is therefore an internal statement, made by a conscious system, and general statements about quantum jumps should be made relative to a particular conscious system.

There is now no problem of arbitrariness in specifying the viable subspaces between which quantum jumps occur. The second law of quantum dynamics, governing the transitions of the visible state, is now to be understood as describing the experiences of conscious systems — it is a collection of different laws, one for each conscious system; and there is no loss of unitary symmetry in the fact that a preferred system of subspaces occurs in each of these laws. There are no preferences in the general statement from which the individual statements of experienced transitions are derived.

There is, however, a preference to be declared for one of two preference-free formulations of the general law governing experienced transitions. In describing the experience of a particular conscious system, the viable subspaces could be determined by means of a particularly relevant set of observables; or they could be the subspaces arising in the Schmidt decomposition relative to a particularly relevant tensor-product split, namely the split into the conscious system and the rest of the world. At first sight the latter possibility looks more attractive, perhaps because of a lingering prejudice that Schmidt decompositions are more canonical than choices of bases. But if we are avowedly describing the experiences of a stated conscious system, this already canonically picks out the basis of states of that system in which it has definite experiences — what Lockwood [21] calls the *consciousness basis*, though I will use the term *experience basis*.³ Moreover, if we were to use the Schmidt decomposition there would be no guarantee that the transitions which the conscious system experiences would actually take it to states of definite experience. It is probably true to a high degree of approximation that the states of the conscious system occurring in the Schmidt decomposition, the eigenstates of its density matrix, will in fact be eigenstates of experience; but this will remain an approximation, and if our aim is to describe experience we should make sure that we actually do so.

We thus arrive at the following general formulation of the laws of motion in quantum mechanics:

1. The universe is described by a time-dependent state vector $|\Psi(t)\rangle$ in the universal state space \mathcal{S} , which evolves according to the Schrödinger

³In passing, let us note that an answer to the question “Why don’t we see superpositions of macroscopic states?” is that there is no experience state describing such seeing. A superposition of two experience states is not an experience state.

equation.

2. The experience of any conscious subsystem C of the universe is described at any time t by a state vector $|\phi_n\rangle$ in the experience basis of that subsystem's state space \mathcal{S}_C . If this experience is described by $|\phi_n\rangle$ at time t , then the probability that it is described by $|\phi_m\rangle$ at time $t + \delta t$ is $T_{mn}\delta t$ where

$$T_{mn} = \frac{\max(J_{mn}, 0)}{\langle \psi_n(t) | \psi_n(t) \rangle}, \quad (11)$$

$$J_{mn} = 2 \operatorname{Im} [\hbar^{-1} (\langle \phi_m | \langle \psi_m(t) | H (| \phi_n \rangle | \psi_n(t) \rangle))] \quad (12)$$

and the states $|\psi_n(t)\rangle$ are the states of the rest of the world, (elements of \mathcal{S}_R where $\mathcal{S} = \mathcal{S}_C \otimes \mathcal{S}_R$), which are the coefficients of the experience basis states in the expansion of the universal state vector with respect to this basis:

$$|\Psi(t)\rangle = \sum_n |\phi_n\rangle |\psi_n(t)\rangle.$$

IS YOUR POSTULATE REALLY NECESSARY?

In the actual practice of physicists giving theoretical analyses of experiments in quantum optics, like the quantum-jump and watched-pot experiments described above, it is quite common to base the analysis on an assumption of quantum jumps — see, for example, [10, 13, 16, 30, 4, 41]. This is sometimes regarded as a practically convenient method of solving the equations of motion for a density matrix, sometimes as a description of a fundamental physical process. If the latter, there is a need for a basic principle to decide exactly what jumps to what; at present [41] this decision is an *ad hoc* one, and different theorists make different assumptions. Attempts to derive a quantum-jump approach from first principles [16, 30], using conventional notions of measurement in quantum mechanics, fall foul of the watched-pot effect; it becomes necessary to restrict the application of the measurement postulates in a way which is once again *ad hoc*. The postulates presented in the previous section are intended to provide a firm basis from which, in principle, definite statements about the occurrence of quantum jumps could be derived.

However, there is another possible attitude. The quantum-jump postulate proposed above is a replacement for the conventional measurement

postulate; both function as interpretations of the time-dependent state vector obtained by solving the Schrödinger equation. This is regarded as a set of different state vectors, one for each time, and each of these is interpreted; thus there is a set of interpretations (one for each time in the postulate proposed here; one for each measurement in the conventional formulation). But one could instead regard the time-dependent state vector as a single object, a vector-valued function of a real variable t , and attempt to give it a single interpretation. Time-dependent phenomena like quantum jumps should then emerge by examining this single interpretation of a time-dependent object. An experiment extended over time should be regarded as a single experiment, in which time-dependent phenomena are registered as a record which can be examined at a later time; a conscious experience extended over time can be treated as a single experience at a later time which contains memories of experiences at different times.

Let us see how such an approach could be applied to Dehmelt's quantum-jump experiment. In the appendix the Schrödinger equation is solved for this system, and it is shown that to a good approximation the solution can be written as

$$\begin{aligned} |\Psi(t)\rangle = & |\psi_0\rangle \left\{ \tilde{f}(t)|0\rangle + \sum_n |\tilde{\phi}_n(t)\rangle \right\} + |\psi_1\rangle \left\{ \tilde{g}(t)|0\rangle + \sum_n |\tilde{\theta}_n(t)\rangle \right\} \\ & + |\psi_2\rangle \left\{ \tilde{h}(t)|0\rangle + \sum_n |\tilde{\chi}_n(t)\rangle \right\}, \end{aligned} \quad (13)$$

where $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle$ are the three atomic states shown in Fig.2, and $|\tilde{\phi}_n\rangle, |\tilde{\theta}_n\rangle$ and $|\tilde{\chi}_n\rangle$ are states of the electromagnetic field, each containing n photons, defined recursively by

$$\begin{aligned} |\tilde{\phi}_n(t)\rangle &= -i\alpha \int_0^\infty \tilde{f}(t-t') e^{-iH_f(t-t')} \left\{ a_R^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_B^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} dt', \\ |\tilde{\theta}_n(t)\rangle &= -i\alpha \int_0^\infty \tilde{g}(t-t') e^{-iH_f(t-t')} \left\{ a_R^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_B^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} e^{-i\Omega_R t'} dt', \\ |\tilde{\chi}_n(t)\rangle &= -i\alpha \int_0^\infty \tilde{h}(t-t') e^{-iH_f(t-t')} \left\{ a_R^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_B^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} e^{-i\Omega_B t'} dt'. \end{aligned} \quad (14)$$

where H_f is the Hamiltonian governing the evolution of the electromagnetic field, and a_R^\dagger and a_B^\dagger are creation operators for the red and blue photons associated with the excited levels of the atom — “emitted when the atom jumps from an excited level to the ground state”, we would say if we had

any warrant for talking about quantum jumps. But now, it could be argued, (13) itself gives us such a warrant. It shows the state at time t as a superposition of states of the atom and the electromagnetic field which can be distinguished by a suitable measurement at time t . Such a measurement could not only determine only how many red and blue photons are present — thus identifying a field state $|\tilde{\chi}_n(t)\rangle$, say — but also, by measuring how far the photons have moved from the atom, measure the value of t' in, for example, $e^{-iH_f(t-t')}a_R^\dagger|\tilde{\theta}_{n-1}(t')\rangle$, i.e. the time at which the last red photon was emitted. Likewise the times of earlier emissions could be determined. In this way the history of the quantum jumps which occurred before time t could be determined by a measurement at time t .

Thus the state $\Psi(t)$ is a superposition of states each of which contains a record of a particular sequence of quantum jumps. The full state $\Psi(t)$ evolves continuously, but each component describes discontinuous events, ultimately because of the quantisation of the electromagnetic field. Explicit postulation of quantum jumps at a fundamental level can be replaced by a single application of the usual measurement postulate at a late time t .

There is probably no conclusive argument against this position, and I suspect that many physicists will find it congenial. Nevertheless, it strikes me as unsatisfactory. It requires an arbitrary choice of the time t at which the all-deciding measurement is to be made; this can be put off to a vague “at the end of the day”, but it remains an instant in time, and nothing can be said about the thereafter. Such an interpretation is an attempt to step outside time and take an eternal view, as is certainly possible in classical physics; but it appears not to be allowed by quantum mechanics. If the interpretation of quantum mechanics is going to be bound by time eventually, it seems to me to be best not to put it off and to acknowledge the inescapable importance of time in our experience and our understanding.

Appendix: Quantum Theory of Quantum Jumps

In this appendix we give a conventional quantum-mechanical theory of the quantum-jump experiment [25, 31, 8], solving the full Schrödinger equation for the system consisting of the three-level atom and the electromagnetic field. This does not appear to have been done in previous theoretical treatments of the experiment. Most authors [1, 19, 20, 27, 28, 32] provide this by considering the atomic density matrix, including off-diagonal elements, and using optical Bloch equations. This, however, requires an assumption that quantum jumps occur in the crucial spontaneous emission part of the experiment. The same is true of the treatment [12] in terms of the state

vector of the atom. A complete treatment of the atom-field system has been given by Zoller, Marte and Walls [43] who obtain an equation for the atomic density matrix in which an effective Hamiltonian, incorporating the effects of quantum jumps in the same way as in the other papers, is justified by an appeal to Mollow's fully quantum-mechanical treatment of resonance fluorescence in two-level atoms [23], but no such treatment has been given for the three-level atom involved in Dehmelt's experiment. Barchielli [3] has given a theory of the experiment in terms of a quantum stochastic equation. He obtains this as an approximation to the deterministic Schrödinger equation of the atom-field system, using the same approximations as in the calculation given here. He does not comment on the origin or physical significance of the stochastic elements in his treatment.

The theory given here may appear to be less than fully quantum-mechanical in that the laser beams are described by classical oscillating fields, but this has been shown by Moller [23] to be an accurate description of a coherent quantum field in interaction with an atom with appropriate boundary conditions at $t = -\infty$. In order to obtain a solution we need to use the rapid-dispersal approximation, which can be justified from quantum electrodynamics [36], and to neglect rapidly oscillating quantities, which is a self-consistency requirement, justified retrospectively.

The atom is illuminated by two light beams, the red beam tuned near the frequency $\Omega_R = (E_1 - E_0)/\hbar$ of the strong transition, and the blue beam tuned near the frequency $\Omega_B = (E_2 - E_0)/\hbar$ of the weak transition to the shelf state. We take these both to be coherent (laser) beams. Let Δ_R and Δ_B be their detunings, so that they are described by classical fields $\mathbf{E}_R \cos[(\Omega_R + \Delta_R)t]$ and $\mathbf{E}_B \cos[(\Omega_B + \Delta_B)t]$. The corresponding contributions to the atomic Hamiltonian are

$$H_R = e\mathbf{r} \cdot \mathbf{E}_R \cos[(\Omega_R + \Delta_R)t] \quad \text{and} \quad H_B = e\mathbf{r} \cdot \mathbf{E}_B \cos[(\Omega_B + \Delta_B)t]$$

but the effect of neglecting rapidly oscillating terms can be obtained by replacing these by

$$H_R = \lambda e^{-i(\Omega_R + \Delta_R)t} |\psi_1\rangle \langle \psi_0| + \lambda^* e^{i(\Omega_R + \Delta_R)t} |\psi_0\rangle \langle \psi_1| \quad (15)$$

$$\text{and} \quad H_B = \Lambda e^{-i(\Omega_B + \Delta_B)t} |\psi_2\rangle \langle \psi_0| + \Lambda^* e^{i(\Omega_B + \Delta_B)t} |\psi_0\rangle \langle \psi_2| \quad (16)$$

where $|\psi_0\rangle$, $|\psi_1\rangle$ and $|\psi_2\rangle$ are the three atomic states and

$$\lambda = e\langle \psi_1 | \mathbf{r} \cdot \mathbf{E}_R | \psi_0 \rangle, \quad \Lambda = e\langle \psi_2 | \mathbf{r} \cdot \mathbf{E}_B | \psi_0 \rangle.$$

The interaction between the atom and the electromagnetic field, which is responsible for spontaneous emission from the excited states $|\psi_1\rangle$ and $|\psi_2\rangle$,

is described by a Hamiltonian of the form [36]

$$H_{\text{int}} = \alpha \left(|\psi_1\rangle\langle\psi_0|a_{\text{R}} + |\psi_0\rangle\langle\psi_1|a_{\text{R}}^\dagger \right) + \beta \left(|\psi_2\rangle\langle\psi_0|a_{\text{B}} + |\psi_0\rangle\langle\psi_2|a_{\text{B}}^\dagger \right) \quad (17)$$

where a_{R}^\dagger , a_{B}^\dagger , a_{R} and a_{B} are single-photon creation and annihilation operators which create and annihilate photons associated with the $1 \rightarrow 0$ and $2 \rightarrow 0$ transitions (we will call these “red” and “blue” photons, although the photon states created by a_{R}^\dagger and a_{B}^\dagger do not have definite frequencies [36]).

The full Hamiltonian for the atom-field system is

$$H = H_{\text{a}} + H_{\text{f}} + H_{\text{R}} + H_{\text{B}} + H_{\text{int}} \quad (18)$$

where H_{a} is the Hamiltonian for the atom alone, which has eigenstates $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle$ with eigenvalues E_0, E_1, E_2 ; and H_{f} is the Hamiltonian for the field alone. The state of the whole system at time t can be written

$$\begin{aligned} |\Psi(t)\rangle = & e^{-iE_0t/\hbar} |\psi_0\rangle \left\{ f(t)|0\rangle + \sum_{n=1}^{\infty} |\phi_n(t)\rangle \right\} \\ & + e^{-iE_1t/\hbar} |\psi_1\rangle \left\{ g(t)|0\rangle + \sum_{n=1}^{\infty} |\theta_n(t)\rangle \right\} e^{-i\Delta_{\text{R}}t} \\ & + e^{-iE_2t/\hbar} |\psi_2\rangle \left\{ h(t)|0\rangle + \sum_{n=1}^{\infty} |\chi_n(t)\rangle \right\} e^{-i\Delta_{\text{B}}t} \end{aligned} \quad (19)$$

where $|0\rangle$ is the vacuum field state and $|\phi_n(t)\rangle, |\theta_n(t)\rangle, |\chi_n(t)\rangle$ are n -photon states. The Schrödinger equation for $|\Psi(t)\rangle$, with the Hamiltonian (5), gives

$$\begin{aligned} i\frac{df}{dt} &= \lambda^*g + \Lambda^*h, \\ i\frac{dg}{dt} &= \lambda f + \Delta_{\text{R}}g + \alpha e^{i\Omega_{\text{R}}t} \langle 0|a_{\text{R}}|\phi_1(t)\rangle, \\ i\frac{dh}{dt} &= \Lambda f + \Delta_{\text{B}}h + \beta e^{i\Omega_{\text{B}}t} \langle 0|a_{\text{B}}|\phi_1(t)\rangle; \end{aligned} \quad (20)$$

$$\begin{aligned} i\frac{d}{dt}|\phi_n\rangle &= H_{\text{f}}|\phi_n\rangle + \lambda^*|\theta_n\rangle + \Lambda^*|\chi_n\rangle \\ &\quad + \alpha e^{-i\Omega_{\text{R}}t} a_{\text{R}}^\dagger |\theta_{n-1}\rangle + \beta e^{-i\Omega_{\text{B}}t} a_{\text{B}}^\dagger |\chi_{n-1}\rangle, \\ i\frac{d}{dt}|\theta_n\rangle &= (H_{\text{f}} + \Delta_{\text{R}})|\theta_n\rangle + \lambda|\phi_n\rangle + \alpha e^{i\Omega_{\text{R}}t} a_{\text{R}}|\phi_{n+1}\rangle, \\ i\frac{d}{dt}|\chi_n\rangle &= (H_{\text{f}} + \Delta_{\text{B}})|\chi_n\rangle + \Lambda|\phi_n\rangle + \beta e^{i\Omega_{\text{B}}t} a_{\text{B}}|\phi_{n+1}\rangle, \end{aligned} \quad (21)$$

with the understanding that $|\theta_0(t)\rangle = g(t)|0\rangle$ and $|\chi_0(t)\rangle = h(t)|0\rangle$.

We can now use the fact that the spontaneously emitted photons quickly move away from the vicinity of the atom to eliminate the terms in $|\phi_1\rangle$ from eqs. (20) and the terms containing $|\phi_{n+1}\rangle$ from (21). Let $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be an

eigenvector of the matrix $A = \begin{pmatrix} 0 & \lambda^* & \Lambda^* \\ \lambda & \Delta_R & 0 \\ \Lambda & 0 & \Delta_B \end{pmatrix}$ with eigenvalue ξ ; then eqs. (21) can be solved for the combination

$$|\Xi_n(t)\rangle = x^*|\phi_n(t)\rangle + y^*|\theta_n(t)\rangle + z^*|\chi_n(t)\rangle \quad (22)$$

as an integral over the interval $0 < t' < t$ of an integrand containing $a_R^\dagger|\theta_{n-1}(t')\rangle$, $a_B^\dagger|\chi_{n+1}(t')\rangle$, $a_R|\phi_{n+1}(t')\rangle$ and $a_B|\phi_{n+1}(t')\rangle$. The last two can be removed by applying an annihilation operator a_B or a_R and using the rapid-dispersal approximations [36]

$$\int_0^t a_X e^{-iH_f(t-t')/\hbar} a_Y |\phi_{n+1}(t')\rangle dt' = 0, \quad (23)$$

$$a_X e^{-iH_f(t-t')/\hbar} a_Y^\dagger |\Upsilon(t')\rangle = 0 \quad \text{if } t - t' \gg \tau, \quad (24)$$

$$\int_0^t a_X e^{-iH_f(t-t')/\hbar} a_Y^\dagger |\Upsilon(t')\rangle e^{-i\Omega_Y t'} F(t') dt' = Q_{XY} |\Upsilon(t)\rangle F(t) e^{-i\Omega_Y t} \quad (25)$$

where X and Y stand for R or B, Q_{XY} is a constant, τ is the dispersal time which is of the order of 10^{-18} s, $|\Upsilon(t)\rangle$ is any of the n -photon states of interest, and $F(t)$ is any slowly varying function. This yields

$$a_R |\Xi_n(t)\rangle = -i\alpha x^* (Q_{RR} |\theta_{n-1}(t)\rangle e^{-i\Omega_R t} + Q_{RB} |\chi_{n-1}(t)\rangle e^{-i\Omega_B t}) \quad (26)$$

and a similar equation with a_B . Note that the right-hand side does not contain the eigenvalue ξ . We can repeat this for each of the eigenvectors $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ of the matrix A ; then, using the fact that these eigenvectors satisfy

$$\mathbf{x}_1 \mathbf{x}_1^\dagger + \mathbf{x}_2 \mathbf{x}_2^\dagger + \mathbf{x}_3 \mathbf{x}_3^\dagger = \mathbf{1},$$

we obtain

$$a_R |\phi_{n+1}(t)\rangle = -i\alpha (Q_{RR} |\chi_n(t)\rangle e^{-i\Omega_R t} + Q_{RB} |\chi_n(t)\rangle e^{-i\Omega_B t}). \quad (27)$$

In particular, for $n = 0$ we obtain expressions for $\langle 0|a_R|\phi_1\rangle$ and $\langle 0|a_B|\phi_1\rangle$ in terms of $g(t)$ and $h(t)$. Putting these into eq. (20) and ignoring rapidly oscillating terms with the frequency $\Omega_B - \Omega_R$ gives a set of homogeneous equations for the amplitudes f, g, h which are the same as those obtained by intuitive arguments assuming quantum jumps:

$$\begin{aligned}\frac{df}{dt} &= -i\lambda^*g - i\Lambda^*h, \\ \frac{dg}{dt} &= -i\lambda f - (\gamma_R + i\Delta_R)g, \\ \frac{dh}{dt} &= -i\Lambda f - (\gamma_B + i\Delta_B)h\end{aligned}\tag{28}$$

where

$$\gamma_R = \frac{\alpha^2 Q_{RR}}{\hbar^2} \quad \text{and} \quad \gamma_B = \frac{\alpha^2 Q_{BB}}{\hbar^2}$$

are the (complex) decay rates (including energy shifts) for the spontaneous transitions $1 \rightarrow 0$ and $2 \rightarrow 0$.

In general, using (27) to eliminate $|\phi_{n+1}\rangle$ from the equations (21) for the n -photon states $|\phi_n\rangle, |\theta_n\rangle, |\chi_n\rangle$ yields inhomogeneous equations which have the same matrix as (28) in their homogeneous part, with a source term for $|\phi_n\rangle$ involving the $(n-1)$ -photon states $|\theta_{n-1}\rangle$ and $|\chi_{n-1}\rangle$:

$$\begin{aligned}i\frac{d}{dt}|\phi_n\rangle &= H_f|\phi_n\rangle + \lambda^*|\theta_n\rangle + \Lambda^*|\chi_n\rangle \\ &\quad + \alpha e^{-i\Omega_R t} a_R^\dagger |\theta_{n-1}\rangle + \alpha e^{-i\Omega_B t} a_B^\dagger |\chi_{n-1}\rangle, \\ i\frac{d}{dt}|\theta_n\rangle &= \lambda|\phi_n\rangle + (H_f + \Delta_R - i\gamma_R)|\theta_n\rangle \\ i\frac{d}{dt}|\chi_n\rangle &= \Lambda|\phi_n\rangle + (H_f + \Delta_B - i\gamma_B)|\chi_n\rangle\end{aligned}\tag{29}$$

(after dropping rapidly oscillating terms).

Suppose the atom is in its ground state at $t = 0$, no photons having been emitted; then the initial conditions are $f(0) = 1$, $g(0) = h(0) = 0$ and $|\phi_n(0)\rangle = |\theta_n(0)\rangle = |\chi_n(0)\rangle = 0$. The solution $(f(t), g(t), h(t))$ of the homogeneous equations (28) with these initial conditions can then be used as a Green's function for the inhomogeneous equations (29) (after taking account of the field Hamiltonian H_f by multiplying by $e^{-iH_f t}$). The result is

$$\begin{aligned}|\phi_n(t)\rangle &= -i\alpha \int_0^t e^{-iH_f(t-t')} f(t-t') \\ &\quad \left\{ a_R^\dagger |\theta_{n-1}(t')\rangle e^{-i\Omega_R t'} + a_B^\dagger |\chi_{n-1}(t')\rangle e^{-i\Omega_B t'} \right\} dt'\end{aligned}\tag{30}$$

with similar expressions for $|\theta_n(t)\rangle$ and $|\chi_n(t)\rangle$. It is more meaningful to present these in terms of the actual n -photon states, without the exponential factors that were introduced into the definition (19) in order to simplify the calculations. Thus, if we define quantities with a tilde so that the full state at time t is

$$|\Psi(t)\rangle = |\psi_0\rangle \left\{ \tilde{f}(t)|0\rangle + \sum_n |\tilde{\phi}_n(t)\rangle \right\} + |\psi_1\rangle \left\{ \tilde{g}(t)|0\rangle + \sum_n |\tilde{\theta}_n(t)\rangle \right\} \\ + |\psi_2\rangle \left\{ \tilde{h}(t)|0\rangle + \sum_n |\tilde{\chi}_n(t)\rangle \right\}, \quad (31)$$

then the n -photon states $|\tilde{\phi}_n\rangle$, $|\tilde{\theta}_n\rangle$ and $|\tilde{\chi}_n\rangle$ are given by

$$|\tilde{\phi}_n(t)\rangle = -i\alpha \int_0^\infty \tilde{f}(t-t') e^{-iH_{\text{f}}(t-t')} \left\{ a_{\text{R}}^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_{\text{B}}^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} dt', \\ |\tilde{\theta}_n(t)\rangle = -i\alpha \int_0^\infty \tilde{g}(t-t') e^{-iH_{\text{f}}(t-t')} \left\{ a_{\text{R}}^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_{\text{B}}^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} e^{-i\Omega_{\text{R}} t'} dt', \\ |\tilde{\chi}_n(t)\rangle = -i\alpha \int_0^\infty \tilde{h}(t-t') e^{-iH_{\text{f}}(t-t')} \left\{ a_{\text{R}}^\dagger |\tilde{\theta}_{n-1}(t')\rangle + a_{\text{B}}^\dagger |\tilde{\chi}_{n-1}(t')\rangle \right\} e^{-i\Omega_{\text{B}} t'} dt'. \quad (32)$$

As in the case of resonance fluorescence in two-state atoms [36], these expressions have a simple intuitive analysis as superpositions of states in which the atom was in an excited state at some time t' , having previously emitted $n-1$ photons, emitted the n th photon at time t' , and emitted no further photons between times t' and t . (This description, however, does not account for the exponential factors $e^{-i\Omega_{\text{R}} t'}$ and $e^{-i\Omega_{\text{B}} t'}$ in $|\tilde{\theta}_n(t)\rangle$ and $|\tilde{\chi}_n(t)\rangle$.) The rapid-dispersal property of the photons created by a_{B}^\dagger and a_{R}^\dagger ensures that the different states in the superpositions do not interfere with each other, so that the probabilities for the existence of n photons follow classical laws; for example, the probability that there are n photons at time t and the atom is in its red state is

$$\langle \tilde{\theta}_n(t) | \tilde{\theta}_n(t) \rangle = 2 \text{Re } \gamma_{\text{R}} \int_0^t |\tilde{g}(t-t')|^2 \langle \tilde{\theta}_{n-1}(t') | \tilde{\theta}_{n-1}(t') \rangle dt' \\ + 2 \text{Re } \gamma_{\text{B}} \int_0^t |\tilde{g}(t-t')|^2 \langle \tilde{\chi}_{n-1}(t') | \tilde{\chi}_{n-1}(t') \rangle dt' \quad (33)$$

which is the probability that would be obtained by a naive argument assuming that the atom is always in one of its three states, that when it is in one of the two higher states it has a probability $(2 \text{Re } \gamma) dt$ of decaying into

the ground state in time dt , and that if it is in the ground state at time t' the probability that it will be in the first excited state at a later time t is $|\tilde{g}(t - t')|^2$.

Eqs. (32) contain the full paradox of the description of time development in quantum mechanics. They represent the development of the system as being continuous in time, and yet they describe an experience (that of watching the trapped atom) which is undeniably discontinuous. This emphasises the shadowy status of the state vector, whose probabilistic and ontological aspects coexist uneasily. Insofar as the state vector has the nature of a probability, it is not surprising that it should change continuously while describing a discontinuous process; this is normal for a discrete stochastic process. In such a process, however, the states are described quite separately from the probabilities. The formulation of quantum mechanics suggested in the body of the paper can be seen as restoring this separation in the distinction between the pilot state and the visible state, the former being part of the statement of probabilities while the latter is (in classical terms) a state specification.

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